IC/94/280

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

SUPERSYMMETRIC MAGNETIC MOMENTS SUM RULES AND SPONTANEOUS SUPERSYMMETRY BREAKING

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ABSTRACT

In supersymmetry the anomalous magnetic moment of particles belonging to the same supermultiplet is related by simple sum rules. We study the modification of these sum rules in the case of the spontaneously broken N=1 global supersymmetry.

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September 1994

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1 Introduction

It was shown [1] that N=1 supersymmetry gives rise to model independent sum rules relating the magnetic transitions between states of different spin with a given charged massive multiplet of arbitrary spin. Denoting the gyromagnetic ratio g_j of a given spin j particle as defined by

$$\boldsymbol{\mu}_j = \frac{e}{2M} g_j \mathbf{J}$$

These rules reduce to $g_{1/2} = 2$ for chiral multiplets and to

$$g_{1/2} = 2 + 2h, \qquad g_1 = 2 + h \tag{1}$$

for vector multiplets, where h is a real number characterizing the magnetic transition between the spin-0 and spin-1 states.

The relevant question for phenomenological implication is how these sum rules modify in the physical situation where the supersymmetry is broken. A first attempt in this direction was accomplished in the work of Ref.[2] where a class of soft supersymmetry breaking terms were introduced.

In this paper we would like to study the modification of these rules with a theory in which supersymmetry is spontaneously broken. The hope is that the spontaneous nature of supersymmetry breaking can guarantee the survival of some interesting relations among the various transition magnetic moments. This can be seen as an intermediate step for a full comprehension of the problem in a theory where a local supersymmetry is spontaneously broken to a global N = 1 SUSY theory plus a set of SUSY breaking terms.

The most general CP and $U(1)_{e.m.}$ invariant $WW\gamma$ vertex when all particles are on mass shell is [3]

$$M_{\mu\alpha\beta} = ie\{A[2p_\mu g_{\alpha\beta} + 4(q_\alpha g_{\beta\mu} - q_\beta g_{\mu\alpha})] + 2\Delta K_{WW}(q_\alpha g_{\beta\mu} - q_\beta g_{\mu\alpha}) + 4\frac{\Delta Q}{m_W^2} p_\mu q_\alpha q_\beta\} \tag{2}$$

where $p - q$, $p + q$, $2q$ are the momenta of the incoming and outgoing W^+ and of the incoming photon. At the tree level of the Minimal Supersymmetric Standard Model A=1, the anomalous magnetic moments and the electric quadruple moment ΔK_{WW} and ΔQ

are equal to zero. This agrees with the fact that in a renormalizable theory of spin $\frac{1}{2}$ and spin 1 particles, the tree level value of the gyromagnetic ratio is equal to 2 and this is implied by the tree level unitarity [4,5].

Radiative contributions modify these tree level results. In the supersymmetric case it was shown that the existing sum rule holds at any order in perturbation theory implying that:

$$\Delta K_{WW} = a_{\omega_1} = a_{\omega_2} = \Delta K_{WH}. \tag{3}$$

where a_{ω_1} and a_{ω_2} are the anomalous magnetic moments of the charginos w_1 and w_2, $a_{\omega_i} = \frac{g_{w_i}-2}{2}$. They are given by the coefficient of

$$\frac{1}{2m_{w_i}} \bar{w}_i \sigma^{\mu\nu} q_\nu w_i \varepsilon_\mu$$

where $e_{w_1} = -e$, $e_{w_2} = +e$, q and ε are the momentum and the polarization vector of the incoming photon.

ΔK_{WH} is the magnetic transition between the spin 1 and spin 0 state in the vector multiplet. It is characterized by the coefficient of

$$\frac{e}{m_W} \varepsilon^{\mu\nu\rho\sigma} p_\rho q_\sigma \varepsilon_\mu \varepsilon'_\nu$$

where p, ε'_ν , q and ε_μ are the momentum and polarization vector of the incoming W^+ and γ respectively.

A first explicit verification of these rules can be found in Ref.[6] where the equality of ΔK_{WW} and a_{ω_i} was shown in the case of massless ordinary fermions. As for massive fermions two cases have been studied:

1- $m_t >> m_W$, with fixed ratio $(\frac{m_b}{m_t})^2 = r$. In this case the supersymmetric sum rules are satisfied [7] and

$$\Delta K^{ql}_{WW} = a^{ql}_{\omega_1} = a^{ql}_{\omega_2} = \Delta K^{ql}_{WH} = \frac{-g^2}{32\pi^2} F(r) \tag{4}$$

with

$$F(r) = \frac{1}{(1-r)^3} [r^3 + 11r^2 - 13r + 1 - 4r(1+2r)\ln r]. \tag{5}$$

2- $m_b = 0$, with fixed ratio $(\frac{m_W}{m_t})^2 = \alpha$. In this case we have [8]

$$\Delta K_{WW}^{q\tilde{q}l\tilde{l}} = a_{\omega_1}^{q\tilde{q}l\tilde{l}} = a_{\omega_2}^{q\tilde{q}l\tilde{l}} = \Delta K_{WH}^{q\tilde{q}l\tilde{l}} = \frac{-g^2}{32\pi^2} G(\alpha) \tag{6}$$

with

$$G(\alpha) = \frac{2}{\alpha^2}[3\alpha + (3 - 2\alpha)\ln(1 - \alpha)]. \tag{7}$$

In the Minimal Supersymmetric Standard Model (MSSM) with supersymmetry broken explicitly but softly by a universal mass $\tilde{m}$ for all scalar particles, the total contribution of the four quantities has been considered [2]. However, in this case, the sum rule (3) results to be badly broken without any interesting functional relation among the four quantities in Eq.(3). In this note we intend to pursue the same strategy in the case of spontaneous breaking of the global N=1 supersymmetry.

For the sake of argument we will consider a SUSY spontaneous breaking realized a'la Fayet-Iliopoulos in the realization of Ref.[8]. In that model the following mass splitting is obtained:

$$\Delta m_{gauge}^2 = \mu^2, \tag{8}$$

$$\Delta m_{matter}^2 = \frac{1}{4}\mu^2 \tag{9}$$

where Δm_{gauge} and Δm_{matter} denote the mass difference between the fermionic and bosonic components in the vector and scalar multiplet respectively, and μ is the proportional to the v.e.v of the Fayet-Iliopoulos term.

The outline of the paper is as follows. In Section 2, we present the calculation of the anomalous magnetic moment of the muon in the simplified case of spontaneously broken super QED. In Sections 3 and 4, we calculate the anomalous magnetic moment of the spin 1 -gauge boson and spin $\frac{1}{2}$ partner respectively. In Section 5, we discuss the modification of the supersymmetric sum rules in this model, and present our conclusion.

2 The anomalous magnetic moment of the muon

The anomalous magnetic moment of the muon, a_μ, is one of the most precisely known quantities, and, hence, it can be used to look for the physics beyond the standard model.

In N=1 global super QED this quantity was shown to be vanishing at any order in perturbation theory [9].

Here we compute the anomalous magnetic moment of the muon in super QED with spontaneous breaking a'la Fayet, for definiteness we consider the mass splitting of Eq.(8). The graphs which will contribute to the anomalous magnetic moment of a_μ are given in Fig.1.

The first diagram, where the muon and photon are running in the loop, gives a contribution to a_μ equal to $\frac{\alpha}{2\pi}$.

From the second diagram, where smuon and photino are running in the loop, one obtains: B

$$\frac{\alpha}{\pi}\int_0^1 dx\,\frac{x(1-x)(a^2x+a)}{-a^2x^2+3/4x-1} \tag{10}$$

where $a^2=\frac{m_\mu^2}{\mu^2}$ and two cases arise:

When $a\rightarrow 0$, i.e $\mu^2 \gg m_\mu^2$, i.e supersymmetry is broken at a high scale, we find that Eq.(9) gives zero, and the anomalous magnetic moment is given by $\frac{\alpha}{2\pi}$, we recover the QED result.

When $a\rightarrow\infty$, i.e $\mu^2\rightarrow 0$ i.e supersymmetry is unbroken. Eq.(9) gives a value equal to $-\frac{\alpha}{2\pi}$ which cancels the contribution of the first diagram. Then the anomalous magnetic moment is $a_\mu=0$, in agreement with the theorem of Ref.[9].

3 The anomalous magnetic moment of the W-boson

We calculate the one loop contributions to the anomalous magnetic moment of the W^+-boson, ΔK_{WW}, with quark and lepton supermultiplet running in the loop. We have a contribution from two diagrams shown in Fig.2. The results obtained from these graphs are in agreement with [2], provided one considers for sfermion masses the SUSY breaking contribution of Eq.(8). For instance in the top generation these results are

$$\Delta K_{WW}(bbt)=\frac{g^2N_cq_b}{32\pi^2}\int_0^1 dx\,\frac{x^4+x^3(a-b-1)+x^2(2b-a)}{bx+a(1-x)-x(1-x)} \tag{11}$$

$$\Delta K_{WW}(ttb)=-\frac{g^2N_cq_t}{32\pi^2}\int_0^1 dx\,\frac{x^4+x^3(b-a-1)+x^2(2a-b)}{ax+b(1-x)-x(1-x)} \tag{12}$$

$$\Delta K_{WW}(\tilde{b}\tilde{b}\tilde{t}) = -\frac{g^2 N_c q_b}{16\pi^2}\int_0^1 dx\,\frac{(x^3-x^2)(\tilde{b}-\tilde{a}-1+2x)}{\tilde{b}x+\tilde{a}(1-x)-x(1-x)} \tag{13}$$

$$\Delta K_{WW}(\tilde{t}\tilde{t}\tilde{b}) = \frac{g^2 N_c q_b}{16\pi^2}\int_0^1 dx\,\frac{(x^3-x^2)(\tilde{a}-\tilde{b}-1+2x)}{\tilde{a}x+A\tilde{b}(1-x)-x(1-x)} \tag{14}$$

where $a=\frac{m_t^2}{m_W^2}$, $b=\frac{m_b^2}{m_W^2}$, $\tilde{a}=\frac{m_{\tilde{t}}^2}{m_W^2}$, $\tilde{b}=\frac{m_{\tilde{b}}^2}{m_W^2}$ and N_c is the number of colours.

The contribution of the leptons of that generation can be obtained from Eq.(10), and of the slepton from Eq.(12). In the case of massless fermions the above equations yield

$$\Delta K_{WW}^{q\tilde{q}l\tilde{l}} = -\frac{4g^2}{16\pi^2}\int_0^1 dx\,\frac{fx(2x-1)}{f-x(1-x)} \tag{15}$$

where $f=\frac{1/4\mu^2}{m_W^2}$. We will examine the value of this integral for two cases:

1- $\mu = M_W$, i.e f= 1/4 and we found $\Delta K_{WW}^{q\tilde{q}l\tilde{l}} = -\frac{4g^2}{16\pi^2}[1/2-\frac{1}{4}\pi]$.

2- $\mu >> M_W$ we found $\Delta K_{WW}^{q\tilde{q}l\tilde{l}} = -\frac{4g^2}{16\pi^2}[1/6]$.

4 The anomalous magnetic moment of the gauginos

We calculate the one loop contribution of the spin 1/2 supersymmetric partners of the W-bosons "ω_1, ω_2". We have a contribution from four diagrams shown in Fig.3. The results of these graphs are in agreement with [2] when taking $m_w^2 = m_W^2+\mu^2$, and they are

$$a_{\omega_1}(b\tilde{t}\tilde{t}) = \frac{g^2 N_c q_t}{16\pi^2}\int_0^1 dx\,\frac{x(x-1)[B(x-2)+x]}{\tilde{A}x+B(1-x)-x(1-x)} \tag{16}$$

$$a_{\omega_1}(bb\tilde{t}) = \frac{g^2 N_c q_b}{16\pi^2}\int_0^1 dx\,\frac{x^2[B(x+1)+x-1]}{Bx+\tilde{A}(1-x)-x(1-x)} \tag{17}$$

$$a_{\omega_1}(t\tilde{b}\tilde{b}) = \frac{g^2 N_c q_b}{16\pi^2}\int_0^1 dx\,\frac{x^2\tilde{B}(1-x)}{\tilde{B}x+A(1-x)-x(1-x)} \tag{18}$$

$$a_{\omega_1}(tt\tilde{b}) = \frac{g^2 N_c q_t}{16\pi^2}\int_0^1 dx\,\frac{x^2\tilde{B}(1-x)}{Ax+\tilde{B}(1-x)-x(1-x)} \tag{19}$$

where

$$A = \frac{m_t^2}{m_{\omega_1}^2} = \frac{m_t^2}{m_W^2+\mu^2} \tag{20}$$

$$\tilde{A} = \frac{m_{\tilde{t}}^2}{m_{\omega_1}^2} = \frac{m_t^2 + 1/4\mu^2}{m_W^2 + \mu^2} \tag{21}$$

$$B = \frac{m_b^2}{m_{\omega_1}^2} = \frac{m_b^2}{m_W^2 + \mu^2} \tag{22}$$

$$\tilde{B} = \frac{m_{\tilde{b}}^2}{m_{\omega_1}^2} = \frac{m_b^2 + 1/4\mu^2}{m_W^2 + \mu^2} \tag{23}$$

if we consider the case of the massless fermions, we find

$$a_{\omega_1}^{q\tilde{q}l\tilde{l}} = a_{\omega_2}^{q\tilde{q}l\tilde{l}} = \frac{2g^2}{16\pi^2}\int_0^1 dx\,\frac{(1+r)x(2x-1)}{r-x} \tag{24}$$

where

$$r = \frac{1/4\mu^2}{m_W^2 + \mu^2} = \frac{f}{1+4f}$$

If we examine the value of this integral for the two cases in the previous section we find

$$a_{\omega_1}^{q\tilde{q}l\tilde{l}} = a_{\omega_2}^{q\tilde{q}l\tilde{l}} = \frac{2g^2}{16\pi^2}[\frac{-27}{256}\pi + \frac{9(-8+3log(7))}{256}],$$

$$a_{\omega_1}^{q\tilde{q}l\tilde{l}} = a_{\omega_2}^{q\tilde{q}l\tilde{l}} = \frac{2g^2}{16\pi^2}[\frac{-5}{32}\pi + \frac{5(-4+log(3))}{125}]$$

respectively.

5 Concluding remarks

In the case of massless fermions the quark and lepton supermultiplet contribution to ΔK_{WH} vanishes because of the zero Yukawa coupling. Hence if we compare Eqs.(15), (24) and their results in the cases which we have studied with the result for ΔK_{WH} in the massless case, it appears that the sum rule (3) is badly broken without any surviving clear pattern. For example in the realistic case where $\mu >> M_W$ we found that $\Delta KB_{WW}^{q\tilde{q}l\tilde{l}} = -0.041667\frac{g^2}{\pi^2}$ and $a_{\omega_1}^{q\tilde{q}l\tilde{l}} = a_{\omega_2}^{q\tilde{q}l\tilde{l}} = -0.07632\frac{g^2}{\pi^2}$. Obviously, considering Eqs.(11)-(14) and Eqs.(16)-(19), it is clear that the problem becomes even more complicated in the case of massive fermion.

In conclusion, our analysis indicates that even the spontaneous breaking of global supersymmetry completely destroys the sum rule in Eq.(3) without leaving any simple pattern.

The investigation of this problem in the fully realistic case of spontaneously broken N=1 supergravity is left for future investigation.

Acknowledgments

The author would like to thank Professor A. Masiero for his guide and A. Culatti for useful discussions. He would also like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

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FIGURE CAPTIONS

Fig.1 Two diagrams contributing to the anomalous magnetic moment of the muon.

Fig.2 Two diagrams contributing to the anomalous magnetic moment of the W–boson.

Fig.3 Four diagrams contributing to the anomalous magnetic moment of the gaugino.